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A PROBLEM OF GROUP PURSUIT WITH PHASE CONSTRAINTS†

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The necessary and sufficient conditions for the capture of at least one evader are obtained in a problem of the simple pursuit of evaders by a group of pursuers, subject to the condition that the evaders use the same control and do not leave the limits of a polyhedral set.

In the theory of differential games, the problem of pursuit by a group of pursuers and the problem of the evasion of a group of pursuers by a single evader are well known [1–8].‡ The situation of a conflict interaction, when two groups, pursuers and evaders, participate in the game is a natural extension of the above-mentioned problems. The aim of the group of pursuers is to capture a specified number of evaders, while the aim of the group of evaders is the opposite [6–14].

Sufficient conditions for the capture of at least one evader were obtained in [13] in a differential game with many pursuers and evaders, subject to the condition that the evaders use the same control. A special case of the above-mentioned problem was considered in [14]. The necessary and sufficient conditions for the capture of at least one evader were obtained in [15] in the case of simple pursuit subject to the condition that the evaders use the same control and there are no phase constraints.

In this paper, the conditions for the capture of at least one evader are obtained in a problem of simple group pursuit, subject to the condition that the evaders use the same control and do not leave the limits of a convex polyhedral set.

1. FORMULATION OF THE PROBLEM

A differential game involving $n + m$ persons: n pursuers and m evaders, is considered in the space $R^k (k \geqslant 2)$. The laws of motion of each of the pursuers P_i (with a control u_i) and of each of the evaders E_j (with a control v) have the form

$$\dot{x}_i = u_i, \quad \| u_i \| \leqslant 1; \quad \dot{y}_j = v, \ \| v \| \leqslant 1; \quad x_i, y_j, \ u_i, v \in R^k \tag{1.1}$$

When $t = 0$, the initial conditions

$$x_i(0) = x_i^0, \quad y_j(0) = y_j^0, \quad \text{and} \quad x_i^0 \neq y_j^0 \tag{1.2}$$

are specified. Here and henceforth, unless otherwise stated, $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$.

It is additionally assumed that, during the course of the game, the evaders do not leave the limits of the convex polyhedral set

$$D = \{y : y \in R^k, \quad (p_s, y) \leqslant \mu_s, \quad s = 1, \ldots, r\}$$

where $p_1, \ldots, p_r$ are unit vectors and $\mu_1, \ldots, \mu_s$ are real numbers such that Int $D \neq \varnothing$.

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‡See also: PETROV N. N., Simple pursuit with the existence of phase constraints. Leningrad, 1984. Deposited at the All-Union Institute for Scientific and Technical Information (VINITI) 27.03.1984, No. 1684–84.

Suppose $T > 0$ and σ is a certain finite subdivision

$$0 = t_0 < t_1 < t_2 < \ldots < t_q < t_{q+1} = T$$

of the segment $[0, T]$.

Definition 1. We shall can the family of mappings c^l ($l = 0, 1, \ldots, q$) that put the measurable function $v(t)$, which is defined for $t \in (t_l, t_{l+1})$ and is such that

$$\| v(t) \| \leq 1, \quad y_j(t) \in D, \quad t \in [t_l, t_{l+1})$$

in correspondence to the quantities

$$(t_l, x_i(t_l), y_j(t_l), \min_{t \in [0, t_l]} \min_i \| x_i(t) - y_j(t) \|) \tag{1.3}$$

the piecewise-preset strategy V of the evaders E_j corresponding to the subdivision σ.

Note that the actions of the evaders can be treated in the following manner: there is a centre which, according to the quantities (1.3), chooses the same control $v(t)$, $t \in (t_l, t_{l+1})$ for all of the evaders E_j.

Definition 2. We shall call the family of mappings b_i^l ($l = 0, 1, \ldots, q$) that put the measurable function $u_i(t)$, which is defined for $t \in (t_l, t_{l+1})$ and is such that

$$\| u_i(t) \| \leq 1, \quad t \in [t_l, t_{l+1})$$

in correspondence to the quantities (1.3) and the control $v(t)$, $t \in (t_l, t_{l+1})$ the piecewise-preset counter strategy U_i of the pursuers P_i, corresponding to the subdivision σ.

Definition 3. In the game Γ, avoidance of an encounter occurs if, for any $T > 0$, a subdivision σ of the segment $[0, T]$ and a strategy V of the evaders E_j exist such that, for any of the trajectories $x_i(t)$ of the pursuers P_i

$$x_i(t) \neq y_i(t), \quad t \in [0, T]$$

Definition 4. In the game Γ, a capture occurs if a $T > 0$ exists and, for any strategy V of the evaders E_j, piecewise-preset counter strategies U_i of the pursuers P_i, an instant $\tau \in [0, T]$ and a number $s \in \{1, 2, \ldots, m\}$, $r \in \{1, 2, \ldots, n\}$ exist such that

$$x_r(\tau) = y_s(\tau)$$

2. AUXILIARY RESULTS

Definition 5 [16]. The vectors $a_1, a_2, \ldots, a_s$ form a positive basis R^k if, for any $x \in R^k$, positive real numbers $\alpha_1, \alpha_2, \ldots, \alpha_s$ exist such that

$$x = \alpha_1 a_1 + \alpha_2 a_2 + \ldots + \alpha_s a_s$$

Lemma 1. Suppose the vectors $a_1, \ldots, a_s$ form a positive basis. Then, for any vectors $b_l, b_{l+1}, \ldots, b_s (1 \leq l \leq s)$, a $\mu_0 > 0$ exists such that, for all $\mu > \mu_0$,

$$a_1, \ldots, a_{l-1}, b_l + \mu a_l, \ldots, b_s + \mu a_s$$

form a positive basis.

Proof. Suppose the lemma is untrue. Then, vectors $b_l, \ldots, b_s$ and a sequence $\mu_n \to \infty$ exist such that

$$a_1, \ldots, a_{l-1}, b_l + \mu_n a_l, \ldots, b_s + \mu_n a_s$$

do not form a positive basis. A sequence $\{v_n\}$, $\|v_n\| = 1$ therefore exists [16] such that, for all n,

$$(a_r, v_n) \leq 0, \quad r = 1, \ldots, l-1; \quad (a_r + \mu_n^{-1} b_r, v_n) \leq 0, \quad r = l, \ldots, s \tag{2.1}$$

By virtue of the compactness of a unit sphere, it can be assumed that $v_n \to v_0$, $\|v_0\| = 1$. On taking the limit in (2.1), we obtain $(a_r, v_0) \leq 0$ that contradicts the positiveness of the basis [16].

Corollary. Suppose the vectors $a_1, \ldots, a_s$ form a positive basis. Then, for any vectors $b_l, b_{l+1}, \ldots, b_s (1 \leq l \leq s)$, a number $\mu_0 > 0$ exists such that, for all $\mu > \mu_0$, the systems of vectors

$$\{a_1, \ldots, a_{l-1}, b_l + \mu a_l, \ldots, b_s + \mu a_s\}, \quad \{a_1, \ldots, a_{l-1}, a_{\alpha_0}, b_r + \mu a_r, r \neq \alpha_0, r = l, \ldots s\}$$

form a positive basis.

Suppose

$$\lambda(a, v) = [(a, v) + [(a, v)^2 + \|a\|^2 (1 - \|v\|^2)]^{1/2}] / \|a\|^2, \quad \beta(a, v) = (a, v)$$

Lemma 2. Suppose $1 < l \leq s$. The vectors $a_1, \ldots, a_s$ form a positive basis when and only when

$$\min_{\|v\| \leq 1} \max\{\lambda(a_1, v), \ldots, \lambda(a_l, v), \beta(a_{l+1}, v), \ldots, \beta(a_s, v)\} > 0$$

3. SOLUTION OF THE PROBLEM

Instead of system (1.1), (1.2), we consider the system

$$\dot{z}_{i,j} = u_i - v, \quad z_{i,j}(0) = z_{i,j}^0 = x_i^0 - y_j^0 \tag{3.1}$$

We will denote the interior and the convex envelope of the set $X \subset R^k$ by Int$X$ and co$X$ respectively.

We shall henceforth assume that the following condition is satisfied: any k vectors from the set are linearly independent and $y_j^0 \in$ Int D.

$$\{x_i^0 - y_j^0, y_q^0 - y_r^0, q \neq r, p_1, \ldots, p_r\}$$

Theorem 1. Suppose

$$0 \notin \operatorname{Int} \operatorname{co}\{x_i^0 - y_j^0, p_1, \ldots, p_r\} \tag{3.2}$$

Then, in the game Γ, avoidance of an encounter occurs.

Proof. It follows from condition (3.2) that a unit vector v_0 exists such that

$$(x_i^0 - y_j^0, v_0) \leq 0, \quad (p_s, v_0) \leq 0 \quad \text{for all} \quad i, j, s$$

We will specify the strategy V of the evaders E_j as follows: $\sigma = \{0, \infty\}$ and $v(t) = v_0$ for all $t \geq 0$. Then, $y_j(t) = y_j^0 + v_0 t$. Hence

$$(p_s, y_j(t)) = (p_s, y_j^0) + t(p_s, v_0) \leq \mu_s$$

The last equation implies that the evaders do not leave the limits of the set D. Moreover,

$$\|x_i(t) - y_j(t)\| = \|x_i^0 + \int_0^t u_i(\tau) d\tau - y_j^0 - v_0 t\| \geq \|x_i^0 - y_j^0 - v_0 t\| - t =$$

$$= [\|x_i^0 - y_j^0\|^2 + t^2 - 2(x_i^0 - y_j^0, v_0) t]^{1/2} - t > 0 \text{ for all } t > 0$$

Theorem 2. Suppose $n \leq k - 1$. Then, an encounter is avoided in the game Γ.

Proof. Suppose the number

$$d = \min_{p,l} \| y_p^0 - y_l^0 \|, \quad \varepsilon > 0 (\varepsilon < d/12)$$

is such that, for all j,

$$\{z : \| z - y_j^0 \| \leqslant \varepsilon\} \subset D$$

Suppose

$$\delta_s = 3\varepsilon/(s+2), \quad J_\alpha(t_s) = \{i : \| y_\alpha(t_s) - x_i(t_s) \| \leqslant \delta_s\}$$

We will now show that, for each $i \in \{1, \dots, n\}$, no more than one set $J_\alpha(t_s)$ exists such that $i \in J_\alpha(t_s)$.

Suppose $i \in J_\alpha(t_s) \cap J_\beta(t_s)$. Then

$$d \leqslant \| y_\alpha(t_s) - y_\beta(t_s) \| \leqslant \| y_\alpha(t_s) - x_i(t_s) \| + \| x_i(t_s) - y_\beta(t_s) \| \leqslant 2\delta_s < d$$

We will specify the strategy V of the evaders E_j as follows:

$\sigma = \{t_s\}_{s=0}^{\infty}$, $t_0 = 0$, $t_{s+1} = t_s + \varepsilon/(s+2)$, $v(t) = v_s$ for all $t \in [t_s, t_{s+1}]$, where v_s, $\| v_s \| = 1$

satisfies the following system

$$(v_s, x_q(t_s) - y_j(t_s)) \leqslant 0, \quad q \in J_j(t_s), \quad (v_s, y_1(t_s) - y_1^0 v) \leqslant 0$$

We will now prove that this strategy guarantees that an encounter will be avoided. We use the notation

$$\Delta_{ij}(t) = \| x_i(t) - y_j(t) \|$$

1. We will show that $y_j(t) \neq x_i(t)$ for all t. Suppose $t \in [t_s, t_{s+1}]$. Then, if $i \in J_j(t_s)$, we have $\Delta_{ij}(t) > 0$ by virtue of the choice of v_s and, if $i \in J_j(t_s)$, then

$$\Delta_{ij}(t_s) > \delta_s, \quad \Delta_{ij}(t) \geqslant \Delta_{ij}(t_s) - 2(t - t_s) > \delta_s - 2(t_{s+1} - t_s) > 0$$

2. We will show that $y_j(t) \in D$ for all $t \in [t_s, t_{s+1}]$. The conditions

$$y_1(t) \in D, \quad \| y_1(t) - y_1^0 \| \leqslant \varepsilon(s+1)/(s+2)$$

follow from the choice of v_s and previously obtained results [17].

Since

$$\| y_j(t) - y_j^0 \| = \| y_1(t) - y_1^0 \| \text{ for all } t$$

then, by virtue of the choice of ε, we obtain $y_j(t) \in D$. Theorem 2 has been proved.

Theorem 3. Suppose $n \geqslant k$ and

$$0 \in \operatorname{Intco}\{x_i^0 - y_j^0, p_1, \dots, p_r\} \tag{3.3}$$

A capture then occurs in the game Γ.

Proof. If $r = 0$, the proof follows from previously obtained results [16].

Suppose $r = 1$. Condition (3.3) means that the vectors $\{x_i^0 - y_j^0, p_1\}$ form a positive basis. We now consider the minimum positive basis. By virtue of the assumption and a previously obtained result ([8], p. 109, Theorem 1.7), the minimum basis set contains $k + 1$ vectors. If the vector p_1 does not occur in the minimum positive basis, the assertion follows from previously obtained results [15]. Suppose p_1 occurs in the minimum positive basis. We denote the set of indices $i \in \{1, \dots, n\}$ occurring in the minimum positive basis by I. Suppose $J(i) = |j|x_i^0 - y_j^0$ occurs in the minimum positive basis, $J = \bigcup_{i \in I} J(i)$. If $|J| = 1$ the assertion of the theorem follows from previously obtained results [8]. Suppose $|J| \geq 2$. We assume that $I = \{1, \dots, q\}, J = \{1, \dots, l\}$.

Next, suppose $c_\alpha^\beta = y_\alpha^0 - y_\beta^0$. Then, $z_{i\alpha}^0 = z_{i1}^0 + c_1^\alpha$ for all i, α, $\alpha \neq 1$. Hence the set

$$\{z_{i1}^0, i \in I,\ c_1^\alpha, \alpha \in J, \alpha \neq 1, p_1\}$$

forms a positive basis. It can be assumed that this basis is the minimum basis. Since $n \geq k$, then $q < n$. Hence, $q + \alpha - 1 \in \{q + 1, \dots, n\}$ for all $\alpha \neq 1$, $\alpha \in J$.

By virtue of the corollary, the set

$$\{z_{i,1}^0, i \in I,\ z_{q+\alpha-1,1}^0 + \mu c_1^\alpha, \alpha \neq 1, \alpha \in J, p_1\}$$

forms a positive basis for a certain $\mu > 1$. We specific the pursuers' controls as follows ($t \in [0, T]$):

$$u_i(t) = v(t) - \lambda(z_{i,1}^0, v(t)) z_{i,1}^0, \quad i \in I$$

$$u_{q+\alpha-1}(t) = v(t) - \lambda(z_{q+\alpha-1,1}^0 + \mu c_1^\alpha, v(t))(z_{q+\alpha-1,1}^0 + \mu c_1^\alpha), \quad \alpha \in J, \quad \alpha \neq 1$$

The instant T will be indicated below.

From system (3.1), we have

$$z_{i,1}(t) = z_{i,1}^0 h_i(t), \quad i \in I \tag{3.4}$$

$$z_{q+\alpha-1,1}(t) + \mu c_1^\alpha = (z_{q+\alpha-1,1}^0 + \mu c_1^\alpha) h_{q+\alpha-1}(t), \quad \alpha \in J, \quad \alpha \neq 1$$

where

$$h_i(t) = 1 - \int_0^t \lambda(z_{i,1}^0, v(\tau)) d\tau$$

$$h_{q+\alpha-1}(t) = 1 - \int_0^t \lambda(z_{q+\alpha-1,1}^0 + \mu c_1^\alpha, v(\tau)) d\tau, \quad \alpha \neq 1, \ \alpha \in J$$

Since the strategy of the evaders is permissible, then $(p_1, y_j(t)) \leq \mu_1$ for all $t > 0$. Hence

$$\int_0^t (p_1, v(\tau)) d\tau \leq \gamma; \quad \gamma = \max_j(\mu_1 - (p_1, y_j^0))$$

By virtue of Lemma 2, the quantity

$$\delta = \min_{\|v\| \leq 1} \max\{\lambda(z_{i,1}, v),\ i \in I,\ \lambda(z_{q+\alpha-1,1}^0 + \mu c_1^\alpha, v),\ \alpha \neq 1,\ \alpha \in J, (p_1, v)\}$$

is positive. Suppose $T_1(t)$ and $T_2(t)$ are two subsets of the interval $[0, t]$ such that

$$T_1(t) = \{\tau \mid \tau \in [0, t],\ (p_1, v(\tau)) < \delta\}$$

$$T_2(t) = \{\tau \mid \tau \in [0, t],\ (p_1, v(\tau)) \geq \delta\}$$

Then

$$\gamma \geq \int_0^t (p_1, v(\tau)) d\tau = \int_{T_1(t)} (p_1, v(\tau)) d\tau + \int_{T_2(t)} (p_1, v(\tau)) d\tau \geq \delta \mu(T_2(t)) - \mu(T_1(t))$$

On the other hand, $\mu(T_2(t)) + \mu(T_1(t)) = t$ (μ is a Lebesgue measure).

It follows from the last two relations that

$$\mu(T_1(t)) \geqslant (t\delta - \gamma)/(1+\delta)$$

Moreover

$$\sum_i h_i(t) = n - \int_0^t \sum_i \lambda_i(v(\tau))d\tau \leqslant n - \int_0^t M(\tau)d\tau$$

where

$$M(\tau) = \max_i \lambda_i(v(\tau)), \lambda_i(v) = \lambda(z_{i,1}^0, v), \quad i \in I;\ \lambda_{q+\alpha-1}(v) = \lambda(z_{q+\alpha-1,1}^0 + \mu c_1^\alpha, v),$$

$\alpha \neq 1, \alpha \in J$

Further

$$\int_0^t M(\tau)d\tau \geqslant \int_{T_1(t)} M(\tau)d\tau \geqslant \delta\mu(T_1(t)) \geqslant \delta \frac{t\delta - \gamma}{1+\gamma}$$

Hence

$$\sum_i h_i(t) \leqslant n - \delta\frac{t\delta - \gamma}{1+\gamma}$$

It follows from the last inequality that an instant T and a number r exist such that $h_r(T) = 0$. If $r \in I$, then $z_{r,1}(T) = 0$ and, consequently, a capture occurs in the game Γ.

If $h_{q+\alpha_0-1}(T) = 0$ for a certain $\alpha_0 \in J$, $\alpha_0 \neq 1$, then $z_{q+\alpha_0-1.1}(T) = -\mu c_1^{\alpha 0}$.

The inclusion

$$0 \in \operatorname{Intco}\{x_i(T) - y_j(T), i \in I,\ j \in J, p_1\} \tag{3.5}$$

holds.

In fact, from relations (3.4), we have $z_{i1}^0 = z_{i1}(T)/h_i(T)$. Moreover

$$z_{i\alpha}(T) = z_{i1}(T) + c_1^\alpha = z_{i1}(T) + z_{i\alpha}^0 - z_{i1}^0 \text{ for all } \alpha \in J,\ \alpha \neq 1$$

Hence

$$z_{i\alpha}^0 = z_{i\alpha}(T) - z_{i1}(T) + z_{i1}^0 = z_{i\alpha}(T) + \frac{1 - h_i(T)}{h_i(T)} z_{i1}(T) \text{ for all } \alpha \in J,\ \alpha \neq 1$$

According to the condition, the system $\{z_{ij}^0, i \in I, j \in J, p_1\}$ forms a positive basis. Hence, the system

$$\left\{\frac{z_{i1}(T)}{h_i(T)},\ z_{i\alpha}(T) + \frac{1 - h_i(T)}{h_i(T)} z_{i1}(T), \alpha \in I \setminus \{1\}, p_1\right\}$$

also forms a positive basis.

Since $h_i(T) \in (0, 1)$, the system of vectors $\{z_{ij}(T), i \in I, j \in J, p_1\}$ forms a positive basis. Hence, the inclusion (3.5) follows.

Since

$$x_i(T) - y_1(T) = x_i(T) - y_{\alpha_0}(T) + \Delta(T), \quad \Delta(T) = y_{\alpha_0}(T) - y_1(T)$$

the set

$$\{x_i(T) - y_{\alpha_0}(T), \Delta(T),\ x_i(T) - y_j(T),\ i \in I, j \in J \setminus \{1\}, p_1\} \tag{3.6}$$

forms a positive basis. It follows from the condition $z_{q+\alpha_0-1 1}(T) = -\mu c_1^{\alpha 0}$ that

$$x_{q+\alpha_0-1}(T) - y_{\alpha_0}(T) = x_{q+\alpha_0-1}(T) - y_1(T) - \Delta(T) = (\mu - 1)(y_{\alpha_0}^0 - y_1^0)$$

Note that $\Delta(T) = y_{\alpha_0}^0 - y_1^0$.

On replacing the vector $\Delta(T)$ in system (3.6) by the vector $x_{q+\alpha_0-1}(T) - y_{\alpha_0}(T)$, we obtain that the set which is obtained here forms a positive basis. Hence,

$$0 \in \text{Intco}\{x_i(T) - y_j(T),\ j \neq 1, p_1\}$$

Adopting the instant T as the initial instant, we obtain a game in which $m - 1$ evaders now participate. On continuing this process further, we obtain that an instant T_0 exists such that

$$0 \in \text{Intco}\{x_i(T_0) - y_s(T_0), p_1\}$$

for a certain s. Capture now follows from the known results in [8].

Suppose the number $r > 1$ is arbitrary. Since the vectors

$$\{x_i^0 - y_j^0, p_1, \ldots, p_r\}$$

form a positive basis, positive numbers α_{ij}, β_s exists such that

$$0 = \sum_{i,j} \alpha_{ij}(x_i^0 - y_j^0) + \sum_s \beta_s p_s \tag{3.7}$$

We consider the vector

$$p_0 = \beta_1 p_1 + \ldots + \beta_r p_r$$

We will show that

$$\{x_i^0 - y_j^0, p_0\} \tag{3.8}$$

form a positive basis if $p_0 \neq 0$ and that a positive basis is formed by

$$\{x_i^0 - y_j^0\}, \quad \text{if} \quad p_0 = 0.$$

Suppose $p_0 \neq 0$, $x \in R^k$. By virtue of the assumption, any k vectors from the set $\{x_i^0 - y_j^0\}$ are linearly independent. Hence, real numbers $\gamma_1, \ldots, \gamma_k$ and vectors $b_1, \ldots, b_k$ from the set $\{x_i^0 - y_j^0\}$ exist which are such that

$$x = \gamma_1 b_1 + \ldots \gamma_k b_k$$

By virtue of relation (3.7), we obtain

$$x = \gamma_1 b_1 + \ldots \gamma_k b_k + d\left(\sum_{i,j} \alpha_{ij}(x_i^0 - y_j^0) + \sum_s \beta_s p_s\right) = dp_0 + \sum_{i,j} \alpha_{ij}^0 (x_i^0 - y_j^0) \tag{3.9}$$

Taking the number d to be positive and sufficiently large, we obtain that all the coefficients on the right-hand side of equality (3.9) are positive. This fact also means that the vectors (3.8) form a positive basis.

The case when $p_0 = 0$ is treated in a similar way. The theorem is proved.

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